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Heavy Quark Physics *

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Lattice calculations of heavy quark systems provide very good measures of the lattice spacing, a key element in recent determinations of the strong coupling constant using lattice methods. They also provide excellent testing grounds for lattice methods in general. I review recent phenomenological and technical developments in this field.

1. INTRODUCTION

Present and future lattice calculations involving b and c quarks include some of the most important applications of lattice gauge theory to standard model physics. These include the heavy meson decay constants, the $B\overline{B}$ mixing amplitude, and various semileptonic decay amplitudes, which are all crucial in extracting CKM angles from experimental data. They also include the extraction of α_s from the charmonium and bottomonium spectra.

Bound states of heavy quarks and antiquarks (quarkonia) have another crucial role to play in the development of lattice gauge theory: they provide systems in which the estimation of the errors inherent in current lattice calculations can be done in a more reliable and robust way than is possible for the light hadrons. The reason is that the quarks in these systems are relatively nonrelativistic. Coulomb gauge wave functions calculated on the lattice may be used to aid in the estimation of finite volume and finite lattice spacing errors, and of the effects of quenching. We have a much better idea of what to expect in lattice calculations of these systems since potential models may be used to obtain the leading behavior in v^2/c^2.[1]

Chris Sachrajda and I will split the subject of heavy quark physics in these proceedings. His review [3] will concentrate on the part of the subject which involves the weak interactions. Mine will concentrate on the part which does not.

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2. LATTICE FORMULATIONS OF HEAVY QUARKS

When the lattice spacing a is smaller than the Compton wave length of the quark $1/m$, the standard relativistic action of Wilson may be used. Cutoff effects may be removed by taking the cutoff $1/a$ to infinity. The bare lattice action may also be viewed as an effective field theory of QCD at the cutoff scale. Cutoff effects in an effective field theory are removed by adding higher dimension interactions to the bare Lagrangian while keeping the cutoff fixed. To remove the effects of the cutoff to a finite order in a, a finite number of interactions may be added to the bare lattice Lagrangian. When $a\Lambda_{QCD} \ll 1$, perturbation theory may be used to calculate the required coefficients of the new operators.[4] The ability to remove cutoff effects perturbatively will probably be spoiled eventually, perhaps at a small power of a due to effects presently not understood, almost certainly at a relatively large power of a due to instantons.

The dynamical scales in bound states are small compared to the fermion mass in QED and in QCD for the c and b quarks. It is often advantageous in these systems to formulate the field theory nonrelativistically as an expansion in $1/m$ [5-8], keeping the cutoff at or below m. The non-leading terms in nonrelativistic expansions have dimension higher than four. Loop corrections in these effective field theories diverge if the cutoff is removed. Cutoff effects must be removed by adding higher dimension interactions to the Lagrangian or by raising the cutoff to the new

physics scale (m), switching to the relativistic, renormalizable version of the theory, and then taking the cutoff to infinity.

When the kinetic energy of the heavy quark is small compared to the typical interaction energies (as it is in bound states containing a single heavy quark), the kinetic energy may be treated as a perturbation. In this *static approximation* [7, 6], the lowest order fermion action is just

$$\mathcal{L}_{\text{static}} = \phi^* i D_t \phi, \qquad (1)$$

and the unperturbed quark propagator is just the timelike Wilson line. In the general case, including quarkonia, the lowest order potential *and* kinetic terms of the Lagrangian of *Nonrelativistic QCD* (NRQCD) [8, 5] (the terms on the first line of Equation 2) must be included in the unperturbed Lagrangian.

$$\mathcal{L}_{\text{NRQCD}} = \phi^* \{ i D_t + \frac{\mathbf{D}^2}{2m}$$
$$+ \frac{g}{2m} \sigma \cdot \mathbf{B} \} \phi$$
$$+ \cdots \qquad (2)$$

Higher order terms in $\frac{1}{m}$ may be added as perturbations.

In processes such as the semileptonic decay of heavy-light mesons, in which one heavy quark decays into another lighter (but still heavy) quark with a high velocity relative to the first, it is possible, and useful, to formulate the static approximation and nonrelativistic QCD as expansions in the small internal quark momentum around some large, external meson momentum.[9] Lattice implementations of this idea have been proposed [10] and are reviewed by Sachrajda.[3]

2.1. The improvement program for Nonrelativistic QCD

In a recent paper [11], Lepage et al. have systematically examined the improvement program for NRQCD with the goal of reducing systematic sources of error from all sources to under 10%. This program involved the following elements:

1) Since NRQCD has been formulated as a nonrenormalizable effective field theory, cut-off effects are removed not by taking the cut-off to infinity, but by adding additional operators to the

bare Lagrangian (the dots in Eq. 2). The infinite number of possible operators must be ordered according to expected size of their effects on the physics. For heavy-light systems, the operator ordering is simply an expansion in $1/m$, that is, in the dimensions of the operators. For heavy-heavy systems like quarkonia, the expansion is complicated by the presence of large quark velocities which do not fall to zero with the quark mass. Operators with the same dimension (such as $\frac{\mathbf{D}^2}{2m}$ and $\frac{g}{2m} \sigma \cdot \mathbf{B}$) are suppressed in their effects on the physics by different powers of v (by v^2 and v^4, respectively, in this case).

2) Once the operators required for a given accuracy have been established, their coefficients must be determined by requiring that the NRQCD Lagrangian reproduce the Green's functions of ordinary QCD to this accuracy.

3) Discrete forms of the required operators must then be defined. As with light quark actions, finite a errors must be estimated. If necessary, correction operators [4] must be added to the action.

4) The coefficients of the operators are modified by quantum effects. Many corrections have been calculated in mean field theory.[11] The corrections for the quark energy shift, mass renormalization, and wave function renormalization have been calculated in full one-loop perturbation theory.[12] Deviations between the mean field and one loop results are rather small, from 0–10%.

The result is a systematic correction program in v, a, and α_s. The correction operators in v and a may be included directly in the simulation action, or evaluated as perturbations using lattice or potential model wave functions.

2.2. A New Action for Four Component Fermions

Because coefficients of higher terms in the NRQCD Lagrangian such as $\frac{\mathbf{D}^2}{2m}$ are explicit functions of $1/m$, the quantum corrections described in 4) above are also explicit functions of $1/m$. These begin to diverge as ma is reduced below a value of order one, making the nonrelativistic expansion impractical. The Wilson action likewise has been thought to have finite lattice spacing errors of order ma which blow up as ma is

raised above one. Since the masses of the b and c quarks are such that ma is often $O(1)$ at current lattice spacings, calculations of such crucially interesting quantities as the heavy meson decay constants f_B and f_D have often involved awkward interpolations between results in the static approximation and results using Wilson fermions through a region where neither approximation is well behaved.[3] While such an approach is probably workable, it would clearly be desirable to have a method for lattice fermions which did not begin to break down right in the region of interest.

To approach such a method, we consider a lattice version of $\mathcal{L}_{\mathsf{NRQCD}}$ with a few minor modifications. Like Wilson fermions (ψ), the fermions of NRQCD contain four components per site: a two-component quark field (ϕ) and a two-component antiquark field (χ). The bare mass is conventionally omitted in NRQCD calculations, but we are free to leave it in the theory. The usual Dirac coupling between quarks and antiquarks is absent (having been transformed into higher derivative interactions by the Foldy-Wouthhuysen transformation), but we may add back a sufficiently suppressed amount of this interaction without spoiling the theory. We thus consider the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} \;=\;& \phi^*(c_1\,\Delta_t^- + m_0 - \frac{c_2}{2}\sum_i \Delta_i^+\Delta_i^-)\phi_n \\
&+\; c_3\,\varphi^*\sum_i \sigma_i\Delta_i\chi_n \\
&+\; \chi^*(-c_1\,\Delta_t^+ + m_0 - \frac{c_2}{2}\sum_i \Delta_i^+\Delta_i^-)\chi_n \\
&+\; c_3\,\chi^*\sum_i \sigma_i\Delta_i\phi_n. \qquad (3)
\end{aligned}
$$

When $c_1 = 1$ (times a correction factor when $ma \gg 1$), $c_2 = \frac{1}{m}$, and c_3 is negligible, it is a good Lagrangin for NRQCD. The point of writing the NRQCD Lagrangian in this particular form is that the action becomes precisely the standard Wilson action with the choice of parameters $c_1 = c_2 = c_3 = 1$. It is thus possible to adjust the parameters in such a way that as m_0 is reduced, instead of blowing up, the theory turns smoothly into the Wilson theory.

It is illuminating to expand the equation for

Wilson propagators nonrelativistically when the mass is large. After normalizing the fields by $\frac{1}{\sqrt{1-6\kappa}}$ [13] (not $\frac{1}{\sqrt{2\kappa}}$ as is conventional) one may obtain

$$
\begin{aligned}
\delta_{\mathsf{no}} =\;& [-\mathcal{E} + \mathcal{M} + (1 - U_{\mathsf{n},0}^\dagger) \qquad (4) \\
&- \frac{1}{2}\left(\frac{1}{m_0} + \frac{1}{(1+m_0)\,(2+m_0)}\right)\sum_i(\Delta_i)^2]\phi_{\mathsf{n}},
\end{aligned}
$$

where $\mathcal{E}$ is the energy eigenvalue obtained from the transfer matrix and $\mathcal{M} = \mathcal{E}_{p^2=0} = \ln(1+m_0)$. This is a lattice Schrödinger equation not unlike the one obtained from NRQCD, but it has some unusual features. Most important, the two "masses" in the equation, $\mathcal{M} = \ln(1 + m_0)$ and $\frac{1}{M} = \frac{1}{m_0} + \frac{1}{(1+m_0)\,(2+m_0)}$, are completely different. If $\mathcal{M}$ is used to fix the fermion mass when $am \gg 1$, the dynamically more important mass condition $\partial\mathcal{E}/\partial p^2 = \frac{1}{2m}$ will be completely incorrect.

Kronfeld showed in his talk at this conference [14] that the two masses can be put back into agreement with the use of the action

$$
\begin{aligned}
S \;=\;& \sum_n[-\bar\psi_n\psi_n \\
&+\; \kappa_t\bar\psi_n(1-\gamma_0)U_{n,0}\psi_{n+\hat{0}} + h.c. \\
&+\; \kappa_s\sum_i \bar\psi_n(1-\gamma_i)U_{n,i}\psi_{n+\hat{i}} + h.c.] \qquad (5)
\end{aligned}
$$

Thus, it seems that an action closely related to the Wilson action is a member of the class of actions suitable for NRQCD. One can go even further. In NRQCD and in the static approximation, $\mathcal{M}$ plays no dynamical role. It can be ignored, and is conventionally thrown away. This suggests that the standard Wilson action itself can be used when $am > 1$ as long as $\mathcal{M}$ is ignored and $\partial\mathcal{E}/\partial p^2 = \frac{1}{2m}$ is used to fix the quark mass, as is done in NRQCD.

This proposal is obviously correct in free field, where we can calculate the behavior of quark propagators exactly to see that the proposed interpretation makes sense. It is certainly correct in mean field theory, too. Mean field improvement of these fermions, as of Wilson fermions, is simply the absorption of a "mean link" u_0 (see Appendix B) into an effective $\tilde\kappa \equiv u_0\kappa$ and then

proceeding as with free field theory. (A plausible estimate of the mean link in this context is probably $u_0 \sim 1/8\kappa_c$.) It remains to be shown whether the theory is somehow spoiled by renormalization.

Perturbatively, Green functions must be expanded in p^2 and α_s. Each term in the expansion is an explicit function of the quark mass, since the theory must be solved exactly in ma. (The is also the case for the loop corrections of NRQCD.[12] If these functions become singular or badly behaved in some way, the theory could conceivably break down. The one loop perturbative corrections contain all of the ugliest features of Wilson and NRQCD perturbation theory simultaneously, and have only been begun. There is, however, one numerical calculation by El-Khadra [15] indicating that nothing too surprising occurs. The one-loop correction to the local current normalization for Wilson fermions with the naive normalization is [16]

$$\langle\psi|V_4^{loc}|\psi\rangle = \frac{1}{2\kappa(1-0.17g^2)}. \tag{6}$$

The correct normalization with mean field improvement is

$$\langle\psi|V_4^{loc}|\psi\rangle = \frac{1}{(1-\frac{6\kappa}{8\kappa_c})(1-0.06g^2)}. \tag{7}$$

The remaining perturbative correction, $0.06g^2$, becomes an explicit (so far uncalculated) function of m (or κ) in the new formalism which must not become singular if the theory is to make sense.

Fig. 1 shows Eqs. 6 (upper curve) and 7 (lower curve) along with a numerical calculation of the quantity at two values of κ (16^3x32 lattice, $\beta = 5.9$). It can be seen that for this quantity, not only is the unknown function of m not singular, it is approximately equal to 1.

Putting the new action on a secure footing will ultimately require: 1) determination of the bare parameters of the action with mean field theory and full perturbation theory, 2) nonperturbative tests of the perturbative results, and 3) phenomenological tests of the resulting action in calculations of well understood physical quantities. Not much of this program has yet been accomplished. However, as argued above, at large
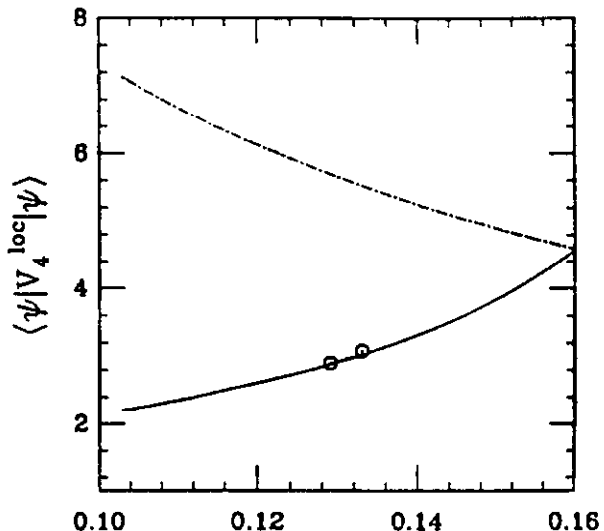

Figure 1. Normalization of the local vector current as a function of κ.

values of ma, the new action (and even the Wilson action suitably reinterpreted) can be viewed simply as unusual members of the general class of lattice actions proposed by Lepage and collaborators for NRQCD. Quite a bit is now known about the action for NRQCD. The discussion in points 1) and 2) of Sec. 2.1 on operator classification is valid for any method for treating heavy quarks, including this one. The fact that the mean field corrections discussed in 4) reproduce the mass-dependent one loop corrections very well is encouraging.

Care will clearly be required in formulating normalization conditions which capture the most important physics in both the relativistic and non-relativistic regions. (Identifying $\partial\mathcal{E}/\partial p^2$ rather than $\mathcal{M}$ as the fundamental mass condition is example number one of these.)

3. PHENOMENOLOGY OF THE J/ψ AND Υ SYSTEMS

Like all phenomenological lattice calculations, calculations of the properties of heavy quark sys-

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tems serve a variety of purposes. Quantities which are well understood experimentally, but which are very sensitive to lattice approximations are good tests of lattice methods (Sec. 3.3). Quantities for which the lattice approximations are well understood may be used to extract information about the standard model (Sec. 3.1). A further purpose for lattice calculations is the delineation of the limits and the reasons for the successes of earlier models of hadrons (Sec. 4.3).

I will discuss calculations in the ψ and Υ systems by Davies, Lepage, and Thacker [17] using NRQCD, and calculations in the ψ system by the Fermilab group using Wilson fermions reinterpreted as described in Sec. 2.2 [18-22], and by UKQCD using Wilson fermions [23]. (See also [24].) Both groups studying the J/ψ system used the $O(a)$ correction term

$$\delta\mathcal{L} = -ig\frac{c}{2}\bar{\psi}\Sigma_{\mu\nu}F_{\mu\nu}\psi \tag{8}$$

of Sheikholeslami and Wohlert [25]. UKQCD used the tree level coefficient $c = 1$. The Fermilab group used a mean field improved coefficient $c = 1.4$ (see Appendix B).

3.1. 1S–1P Splitting

An excellent determination of the lattice spacing in physical units is provided by the spin averaged splitting between the lowest angular momentum ($l = 0$ and $l = 1$) levels of the ψ and Υ systems. (In the charm system, for example. $M_{h_c} - (3M_\psi + M_{\eta_c})/4 = 458.6 \pm 0.4$ MeV.) The values of the lattice spacing obtained from this splitting do not differ dramatically from those obtained from other quantities, such as the ρ mass [26] or the string tension [27]. It is the possibility of making improved uncertainty estimates that makes this an important way of determining the lattice spacing. In quarkonia. error estimates may often be made in several ways: by brute force (e. g., by repeating the calculation several lattice spacings), by phenomenological arguments, and by direct calculation of correction terms. Since determination of the lattice spacing is one of the key components of the determination of the strong coupling constant from low energy physics. it is important that these uncertainty estimates be made rock solid.

Preliminary results for this mass splitting were reported last year by the Fermilab group [18, 19] and by Davies, Lepage, and Thacker [17]. This year, El-Khadra [21] reported further work done to check the corrections and error estimates given in Ref. [18]. In [18], uncertainties due to an incorrectly known quark mass and to $O(a)$ errors arising from an imperfectly determined coefficient c in the $O(a)$ correction to the Lagrangian (Eq. 8) were taken to be less than 1% and omitted from the table of errors on the basis of the phenomenological arguments. (The splitting is expected to be insensitive to small errors in the definition of the quark mass since it is almost identical in the ψ and Υ systems. Likewise. in quark models the contribution of the $\sigma \cdot B$ interaction, which dominates $\delta\mathcal{L}$ nonrelativistically, to the spin averaged splitting is zero.) These arguments were checked this year by repeat calculation at several values of the parameters and found to be correct within statistical errors.

The $O(a^2)$ errors were argued to be small because repeat calculations at $\beta = 5.7$, 5.7, and 6.1 yielded almost the same result for $\alpha(5$ GeV) (see next section). An attempt was made to correct for the small variation observed by extrapolating to zero lattice spacing in a^2. This extrapolation is not completely satisfactory, since the small a functional form is a messy combination of $O(a^2)$ errors and perturbative logarithms. The way to improve this result, which has not yet been done. is to follow the example the NRQCD group [17] and evaluate directly the contributions of the known correction operators to the splitting, thereby eventually obtaining zero measurable dependence on the lattice spacing. This group evaluated the correction of the operators perturbatively using the wave functions of the Richardson [28] potential model. For the Υ at $\beta = 6.0$, for example, they obtained the rather small corrections shown in Table 1. Wave functions directly calculated by lattice gauge theory could also be used, eliminating the need for potential models. They are easy to calculate to high statistical accuracy. Fig. 2 shows the Coulomb gauge wave function of the J/ψ meson calculated on a 24^4 lattice at $\beta = 6.1$. Statisitical errors are negligible at small separations.
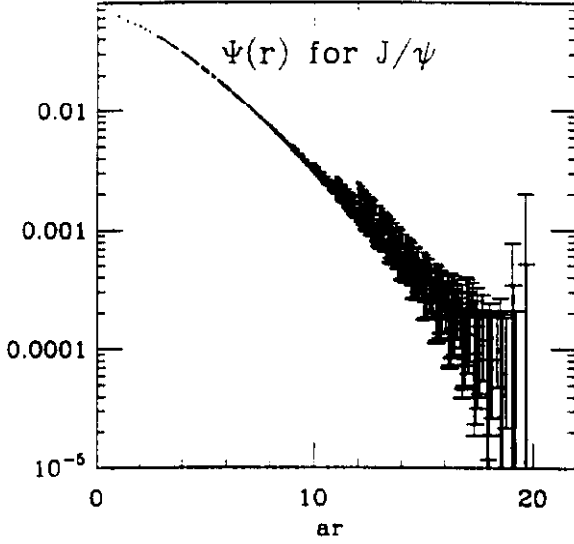
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Figure 2. The wave function of the J/ψ meson.

Term	$\Delta M(1P - 1S)$	%
$O(v^2)$	-11 MeV	-2%
$O(a_t)$	13 MeV	3%
$O(a_s^2)$	-12 MeV	-3%
δS_{gluon}	-24 MeV	-5%
Total	-33 MeV	-7%

Table 1
NRQCD corrections to the 1P–1S splitting in the Υ system. Finite lattice spacing corrections are for $\beta = 6.0$.

Similarly, the estimate of the finite volume correction needs to be bolstered by calculating the meson Coulomb gauge wave functions on the lattice, and then calculating the overlap integral of the wave function with its periodically reflected image.

3.2. Determination of α_s from the 1S–1P Splitting

The most recent determinations of α_s from the charmonium and bottomonium spectra using NRQCD [17] and modified Wilson fermions

[22] have error bars bracketing the region α_s = 0.103 − 0.114 GeV. They are somewhat below, but consistent with the world average given in the review of QCD in the 1992 particle data book. They are inconsistent with the most recent LEP determinations, which are around 0.120 and above.[29]

The determination of α_s from the 1S–1P splitting currently consists of three separate elements: the determination of the lattice spacing, the determination of a physical coupling constant at a scale measured in lattice spacing units, and, for the time being, a correction for the absence of light quarks. As discussed in the previous section, the uncertainties in the determination of α_s arising from the determination of the lattice spacing seem to be in good shape right now, and the path is clear to making them very solid.

3.2.1. Determination of the coupling constant.

To determine the running coupling constant, one would like to combine the determination of the lattice spacing discussed above with a nonperturbative calculation of a physically defined coupling constant, for example defined from the static quark potential at a given, fixed momentum transfer like 5 or 10 GeV.

Since the largest cutoff momenta for the existing 1P–1S splitting calculations was $\pi/a \approx 7.5$ GeV, it was not possible in the existing calculations to determine the continuum limit of a physical coupling defined at short distances.

In Ref. [18] a mean field improved perturbative relation, (Eq. 27 in Appendix B) was used to obtain a renormalized coupling from the bare lattice coupling. This relation was tested over the past year on short distance Wilson loops.[30] It did well, but not perfectly: the loops calculated by Monte Carlo were systematically a few per cent high. This suggests that Eq. 27 fixes most but not all of the pathological relation between the bare lattice coupling constant and physical couplings, and that it is better to obtain physical couplings from short distance quantities calculated nonperturbatively. Using short distance Wilson loops for this purpose (for example, via Eq. 28) raises the values of the renormalized couplings by

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a few per cent over those reported in Ref. [18].

It remains to be determined how much couplings defined from continuum quantities differ from those defined from short distance quantities like the log of the plaquette. There is some reason to expect that this difference is small. The second order corrections to the short distance lattice static potential are within a few per cent of the continuum corrections.[31] Likewise, Creutz ratios of Wilson loops up to six by six are quite well behaved when expanded with a coupling defined from Eq. 28.[30]

The Monte Carlo calculation of the static quark potential at a separation of one lattice spacing agrees to very high accuracy with perturbation using the coupling of Eq. 28. (See Sec. 4.1.) Therefore, a coupling constant obtained from the very short distance static potential will give results almost identical to those using Eq. 28. A phenomenological method for estimating the continuum coupling constant defined by the potential using short distance data has been proposed by Michael.[32] It has been used by UKQCD [33] and by Bali and Schilling.[27] It yields results which are quite close to those obtained with Eqs. 27 and 28, and therefore to those which would be obtained directly from short distance lattice potential itself.

A program to calculate explicitly a continuum coupling constant using finite size scaling has been proposed by Lüscher et al.[34] To select the particular coupling to focus on, they propose the criteria that it: 1) be defined nonperturbatively, 2) be calculable in perturbation theory, 3) be calculable in Monte Carlo simulations, and 4) have small, controllable lattice artifacts. These lead them to propose the response of the QCD vacuum to a constant background color-electric field to define the coupling constant. The more phenomenological choice of the static potential has poorer signal-to-noise properties in Monte Carlo calculations approaching the continuum limit, and (they tell us) more difficult higher order perturbation theory. An SU(2) calculation has been completed, which yields results similar to those obtained with Eq. 27. An SU(3) calculation is in progress.

3.2.2. Correction for the effects of sea quarks.

This is the greatest source of uncertainty in the results quoted above. This correction is the most phenomenological, and has the greatest likelihood of having a problem. Over the next few years, it will be removed by direct inclusion of the effects of sea quarks.

The attempt to estimate the effect that the absence of sea quarks has on this result is based on three assumptions. They are, in order of decreasing rigor:

- When certain physics quantities are used to tune bare parameters in the quenched approximation, the most important terms in the effective Lagrangian at the dominant energy scale for those quantities are given correctly. The effective action at other energy scales including the scale of the lattice cutoff will be somewhat incorrect. In particular, if the effective coupling constant at the physics scale approximates that of the real world, the effective coupling at the short distance cutoff will be a bit small.

- The most important term in the effective action for quarkonia is the static potential. The phenomenological success of potential models indicates that this assumption may be valid to around 25%. It is this assumption, which is certainly not valid for the light hadrons, that leads us to dare to try to make this correction for the charmonium system when we would not try it for the light hadrons.

- The effects of light quarks on the static potential may be estimated by fitting charmonium data with a QCD based potential model such as the Richardson potential once with the correct, $n_f = 3$, β function in the potential and again with the quenched, $n_f = 0$, β function.

The final assumption is certainly a good one at short distances, which are responsible for most of the difference in the evolution of the coupling from the middle distance charmonium physics

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scale down to the lattice cutoff scale. It is also reasonable at the less relevant large distance scale, since the lattice quenched string tension and the string tension of Regge phenomenology are comparable. If, however, light quarks have a much greater effect on the potential in middle distances than they seem to at large and at small distpp ances, the assumption would fail.

The naive expectation for the size of the correction is

$$\frac{\beta_0^{n_f=0} - \beta_0^{n_f=3}}{\beta_0^{n_f=3}} \sim 20\%.$$

In Ref. [18], a perturbative calculation was used to bound the plausible size of the correction. This year the estimated correction was checked [22] by fitting the charmonium spectrum with a potential twice: once using a potential with the correct β function and once using a potential with the quenched β function. (See Sec. 3.3) The result was compatible with the one in Ref. [18]. This, however, is not so much an independent check of the previous estimate as another quantification of the assumption that sea quarks have no more dramatic effects on the potential at middle distances than they seem to at large and small distances.

3.2.3. Future prospects for determining α_s

Errors in the determination of the lattice spacing are already in good shape.

The accuracy in the determination of the coupling constant needs further examination, but the calculations of Ref. [30] suggests that the accuracy to be expected of lattice perturbation theory is greater than that expected of QCD perturbation theory in hadronic phenomenology. In Ref. [30], discrepancies of about α^2 were typically observed in comparisons of first order perturbation theory with Monte Carlo calculations, and discrepancies of about α^3 in comparisons of second order perturbation theory. This amounts to only 3-4% for calculations at the lattice cutoff at moderate β's. In contrast, in QCD phenomenology, an accuracy of 10% is often taken to be optimistic. One difference may be that the lattice calculations of most interest are often quadratically divergent integrals dominated by momenta of the order of the relatively well-defined lattice

cutoff. They thus differ from calculations of unruly hadrons in collision, which insist on interacting on a wide range of momentum scales, piling up large logarithms from a nasty variety of sources.

The aspect of the current determination of α_s which makes it no better than any other existing determination is the use of potential model arguments to estimate the effects of the absence of sea quarks. This is quite analogous to, for example, the phenomenological treatment of higher twist and fragmentation effects in determinations of α_s in deep inelastic scattering. All of the existing determinations have some phenomenological assumptions built into them. The difference is that the potential model estimate of quenched corrections will certainly be eliminated by brute computer force (if not by the use of more intelligent methods) over the next few years, resulting in a determination far more accurate than any of the existing ones.

3.3. Hyperfine Splitting and Leptonic Width

These two quantities are very straightforward to calculate on the lattice, and are good phenomenological tests of how well we understand the parameters of the quark action. The hyperfine splitting $\Delta m(J/\psi - \eta_c)$ and leptonic decay amplitude $V_\psi \equiv m_\psi^2/f_\psi$ have been calculated in the J/ψ system by the Fermilab group and by UKQCD. They have been calculated in the Υ system by Davies, Lepage, and Thacker.[17]

The potential model formula for the hyperfine splitting is [1]

$$\Delta m(J/\psi - \eta_c) = \frac{32\pi\alpha_s(m_c)}{9m_c^2}|\Psi(0)|^2 \ . \tag{9}$$

It arises from a coupling of the spins of the quarks to transverse gluons. It therefore should be extremely sensitive to the value of the correction coefficient c. (It is not clear a priori whether to expect strong sensitivity to the quark mass, since $|\Psi(0)|^2$ should rise with the quark mass.)

The leptonic width to leading order is

$$\Gamma_{ee} = \frac{16\pi\alpha^2 e_c^2}{m_c^2}|\Psi(0)|^2 \ . \tag{10}$$

Nonrelativistically, the leptonic decay amplitude is therefore simply the wave function at the origin,

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$\Psi(0)$, properly normalized. This quantity should be quite sensitive to the mass of the quark.

Before comparing existing lattice results with experiment, we need to estimate the accuracy to expect in the quenched approximation. Both quantities are proportional to $|\Psi(0)|^2$, the probability for the quarks to be at the same point (within one Compton wave length, say) and so are obviously short distance quantities. With lattice parameters tuned to obtain the correct 1P–1S splitting, the coupling constant and potential at short distances will be too weak. An analysis like the one referred to in Sec. 3.2.2 yields

$$\frac{\alpha_s^{(0)}(m_c)}{\alpha_s^{(3)}(m_c)} = 0.81 \pm 0.06 \ . \tag{11}$$

The incorrect weakness of the quenched potential at short distance may also yield a weakened wave function at the origin. El-Khadra has checked for this effect [21, 22] using the Richardson potential

$$V(q^2) = C_F \frac{4\pi}{\beta_0^{(n_f)}} \frac{1}{q^2 \ln\left(1 + q^2/\Lambda^2\right)} \ , \tag{12}$$

where $\beta_0^{(n_f)} = 11 - 2n_f/3$. Fitting the charmonium spectrum with once with $n_f = 3$ in the β function parameter, and again with $n_f = 0$, she found that, indeed, The ratio of the wave functions at the origin was

$$\frac{\Psi^{(0)}(0)}{\Psi^{(3)}(0)} = 0.86 \ . \tag{13}$$

This reduces our expectation for the hyperfine splitting from the experimental result $\Delta m(J/\psi - \eta_c)^{\mathrm{exp}} = 117.3$ MeV to around

$$\Delta m(J/\psi - \eta_c)^{\mathrm{quenched}} \simeq 70 \,\mathrm{MeV} \ . \tag{14}$$

These considerations reduce our expectations for leptonic matrix element by a smaller amount, from the experimental result $V_\psi^{\mathrm{exp}} = 0.509$ GeV$^{3/2}$ to

$$V_\psi^{\mathrm{quenched}} = 0.438 \,\mathrm{GeV}^{3/2} \ . \tag{15}$$

The results of UKQCD and Fermilab for the hyperfine splitting are shown in Fig. 3. UKQCD
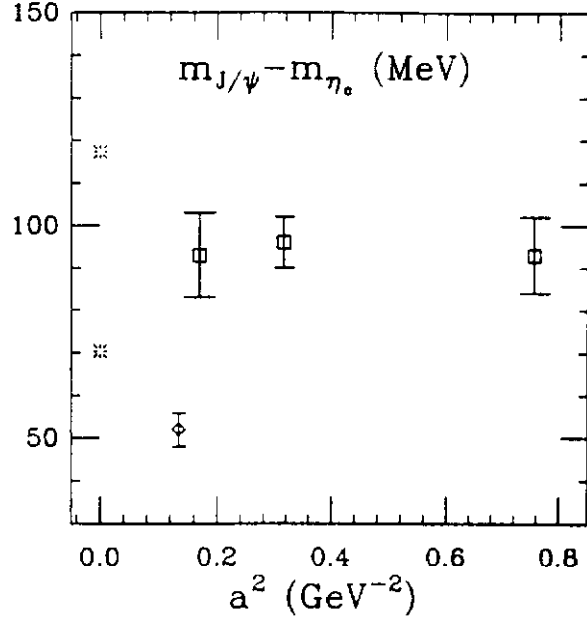

Figure 3. $\Delta m(J/\psi - \eta_c)$ calculated by UKQCD (diamond) and Fermilab (squares) compared with the physical result (upper star) and an estimate of the quenched corrected result (lower star).

set the value of the quark mass to obtain the expected energy eigenvalue in the transfer matrix. In light of the arguments on the interpretation of Wilson fermions at large quark masses in Sec. 2.2, the Fermilab group took this as unreliable and attempted to fix the quark mass by demanding that the leptonic width be correct. The UKQCD result is slightly below the result expected on the basis of the quenched correction. This is consistent with the mean field expectation that quantum corrections boost the required value for the coefficient of the correction term. Their results are, however, much closer to the physical answer than earlier Wilson fermion calculations with no $O(a)$ correction.[35] The Fermilab results are slightly above the quenched expectation, but perhaps not very significantly in light of the uncertainties in the quenched correction and the statistical errors.
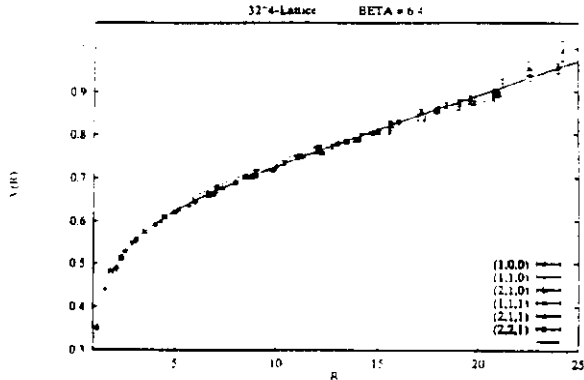
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Figure 4. The heavy quark potential, calculated on the lattice in the quenched approximation.

4. THE STATIC QUARK POTENTIAL

High accuracy results for the static quark potential were reported this year by Bali and Schilling [36, 27] and by UKQCD [33]. Fig. 4 shows the potential calculated by Bali and Schilling in the quenched approximation on a 32^4 lattice at $\beta = 6.4$. The solid line is the fit to a Coulomb plus linear potential

$$V(R) = V_0 - 0.277(28)/R + 0.0151(5)R, \qquad (16)$$

which fits quite well for $R > 2\sqrt{2}$. Comparison of results on 16^4 with results on 32^4 lattice indicated that finite volume results are small. A plot of results from $\beta = 6.0$, 6.2, and 6.4 with physical units set by the string tension indicates good scaling behavior.

4.1. Short distance behavior.

At such a large β we should expect the short distance part of the potential to agree very well with perturbation theory, and this is the case. I checked the value of $V(1)$ given in Ref. [27] against perturbative results for 32^4 lattices supplied by Urs Heller [37]. Using the "measured" coupling constant defined by Eq. 28, perturbation theory agreed with the Monte Carlo data to within about 1%, perhaps fortuitously accurate, but still impressive. (This incidentally illustrates

that the potential is a natural candidate on the lattice as well as in the continuum to define improved coupling constants. The coupling of Eq. 28 was suggested mostly because the plaquette is easy to measure and universally available.)

Since perturbation theory agrees so well with the Monte Carlo calculation of the potential, and since perturbation theory implies a coupling constant rising with increasing R, it would be interesting to attempt to fit the data with an asymptotically free Coulomb plus linear potential. The size of the fit Coulomb term $(0.277(28)/R)$ is quite close to the subleading long distance behavior of the potential $(\pi/(12R) = 0.262/R)$. However these two similarly-sized effects have nothing to do with each other, and we are not guaranteed that, for example, the perturbative Coulomb term does not rise above 0.28 before the potential settles back down to its asymptotic form. A fit with an asymptotically free Coulomb plus linear potential, for example a modified Richardson potential [38], might help to start exploring the extent to which the data support or rule out such speculation.

It is easy to convince yourself with a ruler that values of the string tension obtained by the fit are completely plausible. However, if the changeover from the perturbative Coulomb potential to the nonperturbative long distance $1/R$ term is more complicated than we hope, a larger than expected middle distance Coulomb term could be contaminating the obtained string tensions more than is obvious from the current analysis.

4.2. Long distance behavior.

The good scaling of the potential when the physical scale is set by the string tension has already been mentioned. Good *asymptotic* scaling of the string tension in terms of a physical coupling such as $\Lambda_{\overline{MS}}$ is also observed in the new data. (Good means to perhaps 20%.) Folklore to the contrary was based on the the search for scaling in terms of the bare lattice coupling constant. The bare coupling has a highly pathological, but reasonably well-understood relationship to well-behaved physical coupling constants. It was pointed out long ago [39, 40] that decent scaling is observed in terms of an effective cou-
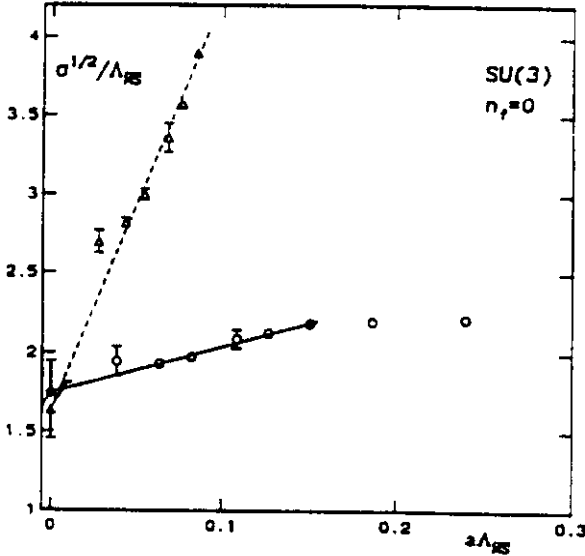
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Figure 5. $\sqrt{\sigma}/\Lambda_{\overline{MS}}$ as a function of the lattice spacing. The upper curve was obtained from the bare coupling constant. The lower curve was obtained from an effective coupling constant.

pling constant defined from the plaquette. It was emphasized in Ref. [30] that such coupling constants are simply very close relations of the familiar physical coupling constants such as α_V and $\alpha_{\overline{MS}}$ of perturbative QCD.

Fig. 5 (from Ref. [40]) shows new and old data for $\sqrt{\sigma}/\Lambda_{\overline{MS}}$ plotted as a function of $a\Lambda_{\overline{MS}}$. The upper curve was obtained via the bare coupling constant. The much better behaved lower curve was obtained via the effective coupling

$$\alpha_{eff} = \frac{3(1 - \frac{1}{3}Tr(U))}{4\pi}. \qquad (17)$$

(Ref. [30] advocates Eq. 28, the logarithm of $Tr(U)$, for this purpose on the grounds that logarithms of Wilson loops have better perturbative behavior than the loops themselves.) Only about 20% deviation from asymptotic scaling is observed over the range of the data. Part of that deviation is certainly perturbative, since the use of another reasonable perturbative scheme, Eq. 28, changes the amount of deviation to 10%.[30]
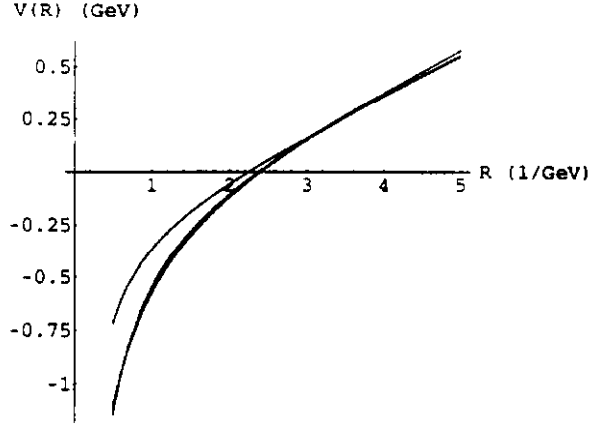

Figure 6. The lattice quenched heavy quark potential (top curve) and the potentials of the Cornell model and the Richardson model.

The fact that the ratio of the deconfinement temperature to the square root of the string tension scales better than $\sqrt{\sigma}/\Lambda_{\overline{MS}}$ is another indication that the deviation is more likely to be connected to the determination of the coupling constant than to the string tension. Because the short distance behavior is a mixture of perturbative logarithms and $O(a^2)$ errors, the extrapolation in a is not completely satisfactory and it is important to sort the origin(s) of the discrepancy: perturbation theory, $O(a^2)$ errors, or measurement errors. However, the downward trend seems clear and the estimate

$$\sqrt{\sigma}/\Lambda_{\overline{MS}} = 1.75 \pm 15\% \qquad (18)$$

seems reasonable.

4.3. Comparison with potential models.

One useful task of first-principles calculations is to support or destroy earlier phenomenologies. For example, it would be nice to be able to understand if there is a reason that nonrelativistic quark models for the light hadrons work unreasonably well. It is more straightforward to put the success of potential models of heavy quark systems on a rigorous footing using lattice methods. These systems are nonrelativistic and it is not

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surprising that a nonrelativistic treatment yields rather accurate results.

In Fig. 6 the potential obtained by Bali and Schilling is compared with the potentials of Eichten et al. [41] and Richarson in the region 0.1 fm $< R <$ 1.0 fm. The string tensions of the lattice and the phenomenological potentials are similar, but the Coulomb term required by phenomenology is about 1.8 times as large as that yielded by the quenched lattice, seemingly a large discrepancy. The phenomenological potential is very well known in this region between 0.1 and 1.0 fm. Fits to the spectra of the charmonium and bottomonium systems with a wide variety of plausible and implausible functional forms yield potentials which differ by only a few per cent in this region. On the other hand, the quenched lattice potentials are also rather convincing, especially at short distance, so what accounts for the difference? First, we expect the quenched Coulomb coupling to be a bit smaller than the true QCD coupling constant at short distances because of the incorrect β function of the quenched approximation. (See Sec. 3.2.2.) This effect is in the right direction and is expected to be of order

$$\frac{\beta_0^{n_f=0} - \beta_0^{n_f=3}}{\beta_0^{n_f=3}} \sim 20\%.$$

Second, the phenomenological potentials clearly parameterize some of the effects of higher order relativistic corrections. These are roughly expected to be of order $v^2/c^2 \sim 25\%$ for charmonium. Some of these clearly have the effect of strengthening the attraction of the quarks, but a complete analysis of the spin-independent relativistic corrections in potential models does not exist.[42] A combination of these two effects could thus easily explain as much as 1.5 out of the discrepancy of 1.8. A preliminary conclusion: there is an interesting puzzle in this discrepancy, but no cause for alarm.

5. SUMMARY

Static Potential.

- At short distances, the potential agrees with perturbation theory to a few per cent.

- The string tension exhibits two loop asymptotic scaling to an accuracy of 20%. $\sqrt{\sigma}/\Lambda_{\overline{MS}}$ is in the range 1.55–1.95.

ψ and Υ systems.

- The hyperfine splitting and leptonic widths provide good phenomenological tests of lattice methods.

- The spin averaged 1P–1S splitting provides a very good determination of the lattice spacing in physical units. Combined with a lattice determination of the renormalized coupling, it gives a determination of the strong coupling constant which at present is of comparable accuracy to that of conventional determinations. When the effects of sea quarks are properly included, its accuracy will be much better than any current determination.

Technical developments.

- Lattice perturbation theory works very well when renormalized coupling constants are used.

- Minor changes to the actions of Wilson and of NRQCD may make it possible do calculations with a unified formalism at any value of the quark mass, as long as the three momentum is small. This will imply a reinterpretation of calculations with Wilson fermions at large quark mass.

ACKNOWLEDGEMENTS

A. NOTATION

We use the forward, backward, and symmetric finite difference operators

$$\Delta_\mu^+ \psi_n \equiv U_{n,\mu}\psi_{n+\hat\mu} - \psi_n, \qquad (19)$$

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$$\Delta_\mu^- \psi_n \equiv \psi_n - U_{n-\hat\mu,\mu}^\dagger \psi_{n-\hat\mu}, \qquad (20)$$

$$\Delta_\mu \psi_n \equiv \frac{\Delta_\mu^+ + \Delta_\mu^-}{2} \psi_n. \qquad (21)$$

The analogous continuum covariant derivative is denoted

$$D_\mu \qquad (22)$$

The standard relation between the bare mass m_0 and the hopping parameter κ is

$$m_0 \equiv \frac{1}{2\kappa} - 4. \qquad (23)$$

When considering nonrelativistic fermions, we decompose the four-component Dirac field as two two-component fields

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \qquad (24)$$

and take as our representation of the Euclidean gamma matrices

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}. \qquad (25)$$

B. RESULTS FROM LATTICE PERTURBATION THEORY

This section summarizes results from Ref. [30] which have been used in the text.

B.1. A sequence of improved coupling constants

In Ref. [43] (1990) it was argued that lattice perturbation series are much more convergent and agree better with Monte Carlo data if they are expressed in terms of a physical running coupling evaluated at a carefully chosen scale. A good one is α_V, the one defined by the static quark potential:

$$\frac{1}{\alpha_V(q)} = \frac{1}{\alpha_{lat}} + \beta_0 \ln(\frac{\pi}{aq}) - 4.702. \qquad (26)$$

The arguments were analogous to those leading from the MS to the $\overline{MS}$ scheme in dimensionally regularized QCD.

In Ref. [18] (1991) it was noted that the bulk of the correction coefficient in the previous equation is accounted for by a simple mean field argument.

The coupling constant is enhanced at one loop by a coupling to the expectation value of the plaquette induced by the higher order terms in the Wilson action. Higher order analogues of this one loop effect certainly exist. This suggests that an effective coupling constant which incorporates a Monte Carlo calculation of the plaquette expectation value, such as

$$\frac{1}{\alpha_V(q)} = \frac{\langle Tr U_p \rangle}{\alpha_{lat}} + \beta_0 \ln(\frac{\pi}{aq}) - 0.513, \qquad (27)$$

may yield improved accuracy.

Over the past year (1992) we have tested this assumption by calculating a variety of short distance quantities using the mean field improved coupling constant and by Monte Carlo.[30] We found that, while using Eq. 27 significantly improved agreement between perturbation theory and Monte Carlo, the Monte Carlo results tended to be systematically slightly higher then the perturbative results. This suggests that a coupling defined directly from any of the Monte Carlo calculated quantities would yield improved predictions for the others. A particularly simple one is the coupling defined from the log of the plaquette:

$$\frac{1}{\alpha_V(3.41/a)} = \frac{1}{\alpha_{eff}} - 1.19$$

$$\equiv -\frac{4\pi}{3\ln(\frac{1}{3}Tr(U))} - 1.19 \qquad (28)$$

(The scale of the running coupling arises from an estimate of the typical momenta of gluons in the calculation of the logarithm of the plaquette.[30])

B.2. Mean field improvement of operators

The mean field argument leading to Eq. 27 may be summarized as follows. The naive classical relation between the lattice and continuum gauge fields

$$U_\mu(x) \equiv e^{iagA_\mu(x)} \rightarrow 1 + iagA_\mu(x). \qquad (29)$$

is spoiled by tadpoles arising from the exponential form of the lattice representation of the gauge fields. Quantum fluctuations do not lead to an average link field close to 1.00000 as implied by Eq. 29, but to something more like

$$U_\mu(x) \rightarrow u_0 (1 + iagA_\mu(x)), \qquad (30)$$

where u_0, a number less than one, represents the mean value of the link. In a smooth gauge, the Monte Carlo link expectation value can be used as an estimate of u_0. A simple, gauge-invariant definition is

$$u_0 \equiv \langle \tfrac{1}{3} \mathrm{Tr} U_{\mathrm{plaq}} \rangle^{1/4}. \tag{31}$$

Other definitions based on κ_c or the static quark self-energy may be used to fine tune mean field predictions in particular situations.

If naive definitions such as Eq. 29 are used to relate lattice and continuum operators, large corrections will appear in quantum corrections. Much better behavior of loop corrections is obtained by taking $U_\mu(x)/u_0$ as the lattice approximation to the continuum field. This implies that the lattice action

$$\tilde{S}_{\mathrm{gluon}} = \sum \frac{1}{\tilde{g}^2 u_0^4} \mathrm{Tr}(U_{\mathrm{plaq}} + \mathrm{h.c.}). \tag{32}$$

will approximate closely the desired continuum behavior. This is the usual lattice action if we identify

$$\tilde{g}^2 = g^2_{lat}/u_0^4 = g^2_{lat}/\langle \tfrac{1}{3} \mathrm{Tr}(U_{\mathrm{plaq}}) \rangle. \tag{33}$$

The perturbative result, Eq. 27, explicitly verifies that $\tilde{g}^2$ is a closer approximation to a standard continuum expansion parameter than g^2_{lat} is.[44]

The same considerations lead to the result that

$$\tilde{\kappa} \equiv \kappa u_0 \tag{34}$$

produces a more continuum-like bare mass ($\tilde{m} = \frac{1}{2\tilde{\kappa}} - 4$) and smaller quantum corrections in operator renormalizations than does κ.

Mean field arguments may be used both to estimate perturbative predictions when the perturbative predictions are unknown, and also to improve known predictions. Just as we did with the coupling constant in Eq. 27, we can improve perturbative predictions for operators involving quark fields by substituting the Monte Carlo calculation of u_0 in Eq. 34 and including in the perturbative prediction only that part remaining after the absorption of u_0 into κ. A possible fine tuning in this case is to obtain u_0 from κ_c rather than from the plaquette. This was the procedure used in obtaining Eq. 7.

The cloverleaf approximations to $F_{\mu\nu}$ used in the Wohlert-Sheikholeslami $O(a)$ correction to the Wilson action [25] and in the magnetic spin coupling of NRQCD [11] contain four links each. The quark wave function normalization contains one link. We therefore expect the naive coefficient of this operator to undergo quantum corrections of roughly a factor of u_0^{-3}, or about $1 + 0.25g^2$ if we use the plaquette to estimate u_0. This is in agreement with an unpublished thesis calculation of Wohlert, $1 + 0.27g^2$.[45] Using the plaquette calculated by Monte Carlo to estimate the correction term yields a factor of $\langle \tfrac{1}{3} \mathrm{Tr}(U_{\mathrm{plaq}}) \rangle^{-3/4} \sim 1.4 - 1.5$ for β around 6.0.

REFERENCES

1   For a review on heavy quarkonia see for example: W. Kwong, J. L. Rosner, C. Quigg, Ann. Rev. Nucl. Part. Sci. 37 (1987) 325: the importance of quarkonia to lattice gauge theory has been emphasized by Lepage, [2].

2   G. P. Lepage, in *Lattice 91*, M. Fukugita et al., editors, Nuc. Phys. B (Proc. Suppl.) 26 (1992) 45.

3   C. T. Sachrajda, in these proceedings.

4   K. Symanzik, Nucl. Phys. B226 (1983) 187, 205.

5   G. P. Lepage and B. A. Thacker, in *Field Theory on the Lattice*, proceedings of the International Symposium, Seillac France, 1987, edited by A. Billoire et al. [Nuc. Phys. B (Proc. Suppl.) 4 (1988) 199]; and Phys. Rev. D 43 (1991) 196.

6   E. Eichten, in *Field Theory on the Lattice*, proceedings of the International Symposium, Seillac France, 1987, edited by A. Billoire et al. [Nuc. Phys. B (Proc. Suppl.)accirac 4 (1988) 170].

7   E. Eichten and F. L. Feinberg, Phys. Rev. Lett. 43 (1979) 1205.

8   W. E. Caswell and G. P. Lepage, Phys. Lett. 167B (1986) 437.

9   N. Isgur and M. B. Wise, Phys. Lett. B232 (1989) 113.

10 J. Mandula and M. Ogilvie, Phys. Rev. D45 (1992) 2183.

11 G.P. Lepage, L. Magnea, C. Nakhleh,

14

U. Magnea, and K. Hornbostel, Cornell preprint CLNS 92/1136, to be published in Phys. Rev. D. See also [2]

12 C. T. H. Davies and B. A. Thacker, Phys. Rev. D45 (1992) 915.

13 This is the normalization used in M. Lüscher, Commun. Math. Phys. 54, (1977) 283.

14 A. S. Kronfeld and P. B. Mackenzie, in preparation; A. S. Kronfeld, contribution to these proceedings.

15 Which she has seen fit to include in the contribution of A. S. Kronfeld to these proceedings.

16 G. Martinelli and Y.-C. Zhang, Phys. Lett. 123B (1983) 433.

17 C. T. H. Davies, G. P. Lepage, and B. A. Thacker, (1991,1992) to be published.

18 A. X. El-Khadra, G. Hockney, A. S. Kronfeld, and P. B. Mackenzie, Phys. Rev. Lett. 69 (1992) 729.

19 P. B. Mackenzie, in *Lattice 91*, M. Fukugita et al., editors, Nuc. Phys. B (Proc. Suppl.) 26 (1992) 369.

20 A. X. El-Khadra, in *Lattice 91*, M. Fukugita et al., editors, Nuc. Phys. B (Proc. Suppl.) 26 (1992) 372.

21 A. X. El-Khadra, contribution to these proceedings.

22 A. X. El-Khadra, G. Hockney, A. S. Kronfeld, and P. B. Mackenzie, to be published.

23 C. R. Allton et al., UKQCD collaboration, Southampton preprint SHEP 91/92-27 (1992).

24 While this review was being completed, a new preprint by UKQCD appeared on the network: S. M. Catterall et al., DAMTP-92-70.

25 B. Sheikholeslami and R. Wohlert, Nucl. Phys. B259 (1985) 572.

26 F. Butler, H. Chen, A. Vaccarino, J. Sexton, and D. Weingarten, contribution to these proceedings.

27 G. S. Bali and K. Schilling, Wuppertal preprint WUB 92-29 (1992).

28 J. Richardson, Phys. Lett. 82B (1979) 272.

29 R. K. Ellis, review given at DPF 92, Fermilab, November 10-14, 1992.

30 G. P. Lepage and P. B. Mackenzie, Fermilab preprint 91/355-T (Revised) (1992).

31 U. Heller and F. Karsch, Nuc. Phys. B251 [FS13] (1985) 254.

32 C. Michael, Phys. Lett. B283 (1992) 103.

33 S. P. Booth et al., UKQCD Collaboration, Liverpool preprint LTH 285 (1992).

34 M. Lüscher, R. Sommer, U. Wolff and P. Weisz, CERN preprint CERN-TH 6566/92, and references therein; talk by M. Lüscher in these proceedings.

35 M. Bochicchio et al., Nuc. Phys. B372 (1992) 403.

36 G. S. Bali and K. Schilling, Wuppertal preprint WUB 92-02 (1992).

37 I thank Urs Heller for these results, obtained with programs written for Ref. [31]

38 The potential used in Ref. [28], modified to make the linear coefficient independent of the Coulomb coefficient may be a reasonable one to try for this purpose.

39 F. Karsch and R. Petronzio, Phys. Lett. 139B (1984) 403.

40 J. Fingberg, U. Heller and F. Karsch, Bielefeld preprint BI-TP 92-26.

41 E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D17 (1978) 3090.

42 As opposed to the analysis of the spin-dependent relativistic corrections by E. J. Eichten and F. Feinberg, Phys. Rev. Lett. 43 (1979) 1205, and Phys. Rev. D 23 (1981) 2724; W. Buchmuller, Phys. Lett. 112B (1982) 479; and D. Gromes, Z. Phys. C 26 (1984) 401.

43 G. P. Lepage and P. B. Mackenzie, in *Lattice 90*, U. M. Heller et al., editors, Nuc. Phys. B (Proc. Suppl.) 20 (1991) 173.

44 Another version of this argument is given by G. Parisi, in *High Energy Physics-1980*, proceedings of the XX International Conference, Madison, Wisconsin, L. Durand and L. G. Pondrom, editors, American Institute of Physics (1981).

45 R. Wohlert, Ph.D. Thesis, unpublished DESY preprint 87/069.